

TENSORIAL APPROACH TO THE GEOMETRY OF BRAIN FUNCTION: CEREBELLAR COORDINATION VIA A METRIC TENSOR

A. PELLIONISZ and R. LLINÁS

Department of Physiology and Biophysics, New York University Medical Center,
550 First Avenue, New York, N.Y. 10016, U.S.A.

Abstract—Based on our previous assumption that the brain must be considered a tensorial entity (PELLIONISZ & LLINÁS, 1979a), a concise model for cerebellar function is proposed: *The cerebellum acts as a metric tensor establishing a geometry for the CNS motor hyperspace.*

This view assumes that the brain is a 'geometrical object', that is to say, (1) activity in the neuronal network is vectorial, and (2) the networks are organized tensorially: i.e. activity vectors remain invariant to changes in reference-frames. Understanding brain functions becomes, then, the establishment of the inherent geometrical properties of the activity vectors and, more fundamentally, the determination of the properties of the multi-dimensional internal space (a frequency hyperspace) in which the vectorial transformations occur. A basic question is the form in which vectors are expressed. Are the components orthogonal projections to an oblique set of coordinate-axes or are they the parallelogram vectorial components? Beyond this, the question arises as to whether the CNS hyperspace is endowed with a geometry determined by a *metric tensor*.

Thus the problem of motor coordination is approached geometrically: in these terms intended movements, i.e. intended movement vectors, are generated in the CNS in reference to the three-dimensional space. These movements are executed, however, by the multiparameter system representing the musculo-skeletal apparatus. Given this increase in dimensionality the question of uniqueness needs to be answered: how are the particular components of a motor vector established if they only represent one choice out of the infinite set of possible vectorial solutions?

The tensorial treatment of motor coordination suggests that the problem is solved by the embedding of the external three-space into the multidimensional CNS space and that this hyperspace is endowed with a metric tensor represented, for movements, by the cerebellar neuronal network. Thus, a unique implementation of the three-dimensional intended movement is possible by a proposed two-step scheme: First, an intended movement vector is specified by an overcomplete number of CNS covariant vector-components. Second, since covariant components cannot be used directly to execute a movement vector (they yield 'dysmetric' movements), they are transformed into contravariant, physical, components of the final motor output. This transformation is the proposed role of the cerebellar metric tensor, resulting in a coordinated unique implementation of movements.

While in this paper the vectorial components transformed by the metric are treated as space coordinates, it has not escaped our attention that the movement space-time is actually four dimensional. This point will be elaborated in a forthcoming paper in which the above concept of metric tensor is expanded to include space-time coordinates. Thus, the predictive feature of cerebellar action together with the metric concept provide a unified approach to the dynamic properties of motor control.

Beyond providing a geometrical model of motor coordination by the cerebellum, the tensorial approach suggests that the 'covariant analysis' and 'contravariant synthesis', via metric tensor, may be a general principle of the organization of the CNS.

IN A RECENT paper we have expressed the general hypothesis that the brain should be regarded as a 'geometrical object' (PELLIONISZ & LLINÁS, 1979a). That is, (1) a system where activity patterns in neuronal networks are to be regarded as vectors, and also (2) where the role of such networks is to generate an internal CNS space in which the vectorial relations are established tensorially. This approach implies that while the neuronal networks of a particular brain are individual, there exists an invariant geometrical property (for the class of all similar input and output relations) that is common for all networks. This is a

formal equivalent of the approach implicit in brain research which aims at understanding *the* brain from studies of individual particular brains.

Thus, given a *particular* neuronal network, its activity is determined by the connectivity between input and output neurons. This network matrix establishes a relation between the patterns of activity (e.g. spiking frequencies) of the input and output neurons. As an example, if a bundle of n nerves could carry light, as in fiber optics, and if action potentials could generate flashes, the input and output patterns (pictures) would be described as vectors in an n -dimensional space. In abstract terms, then the activity pattern of a population of n neurons is defined as an n -dimensional vector, where the n coordinate components are the n