frequencies, and thus the connectivity matrix establishes a transformation of the input vector 'pattern' into an output vector. Obviously it is the physical arrangement of 'wires' in the particular connectivity matrix that determines the input–output transformation. Our fiber optics example also tells us that given diameter irregularities of the fibers one may distort a picture (enlarge or reduce a portion, etc). However, the same distorted picture–picture relation may be obtained by reflecting the input onto a curved mirror, or an appropriate lens. Thus, rather than describing the properties of the different sets of transformations by vectors and matrices (i.e. how points of the picture are carried through the network), the reference-frame invariant tensorial approach can express, in a universal manner, the optical transformation without connectivity matrices.

Indeed, when considering brain function, the realization that activity vectors are expressed via reference-frame invariant *tensorial* entities makes it possible to approach the concept of function beyond the idiosyncratic vector and matrix features of any individual brain circuitry. As a consequence, attention may be shifted from the vector components (expressed in specific coordinates) to the study of vectors themselves, and even more importantly, to the properties of the space of the vectors. In short, in order to describe the global properties of CNS function it is an absolute necessity that the intrinsic geometrical properties of the CNS hyperspace be understood. For example, questions such as whether the brain hyperspace is endowed with a metric tensor immediately arise.

In this paper we point out that motor coordination is intimately related to the geometry of the CNS hyperspace. This approach provides both the necessary concepts as well as the tools to deal with the reference-frame invariant properties of CNS vectors. We hold that useful insights may be obtained not merely by amalgamating the components of the distributed neuronal activities into a single vector, but also by removing these vectors from specific coordinate systems via their tensorial interpretation. This work has been presented in a preliminary form (PELLIONISZ & LLINÁS, 1979*b*).

COORDINATED MOVEMENTS AS TENSORIAL ENTITIES

Movements of the body are physical vectors and as such invariant to reference-frames. This feature, as is also the case with the reference-frame invariant character of physical laws (e.g. that of concerning forces and accelerations), is the basis of the tensorial approach to motor coordination which is implemented in the CNS by the cerebellum.

Consider, for example, a simple limb movement, such as raising an arm. Suppose that a limb is composed of only an upper and lower arm, each of length r, and that it can move only in two dimensions (Fig. 1). Let the hand move with an 'up' displacement vector, $\bar{U}$. Strictly speaking, in this movement we are actually changing the angles α and β at the shoulder and at the elbow. Transforming these two quantities in a particular manner (depending, for example, on the initial limb position) results in the upward displacement vector, $\bar{U}$. Whereas $\bar{U}$ is a physically existing objective entity, the *same* vector is expressed in two fundamentally different kinds of frame of reference. One type can be applied to the external physical space in which the movement is executed while another type of coordinate system refers to the internal multidimensional space in which the movement vector is generated.

Formally, these two different spaces can be incurred by an infinite number of particular reference-frames. For example, considering arm movements restricted to a two-dimensional plane, the limb-displacement is a two-dimensional vector, given by an ordered set of two quantities (x and y, using Cartesian orthogonal rectilinear reference-frame, or R and Ψ, using polar orthogonal curvilinear coordinate-system, and so on). Similarly, the same physical vector can be

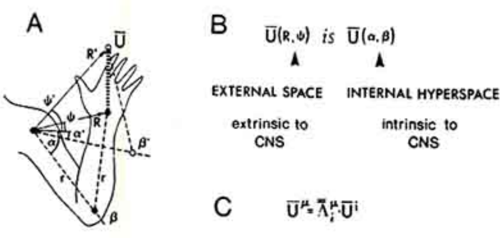

FIG. 1. Limb movement as a tensorial entity. (A) An upward displacement vector $\bar{U}$ is a physical entity which can be expressed in different reference-frames: E.g. by the R, Ψ polar coordinate system, or by the α, β ordered set of two quantities. (B) The two shown reference-frames are of fundamentally different kinds: R, Ψ applies to the CNS-independent external space, the α, β is to the space inherently connected to CNS. The limb-displacement vector occurs in both spaces. (C) Different expressions of the one $\bar{U}$ vector are related by the limb-displacement tensor, $\bar{\Lambda}^{\mu}_{i}$.