For the sake of a quantitative example, let us assume that the length of the arm is $r = 10$ units, and suppose that the intended displacement vector specifies that the hand should move 1 unit farther and 0.2 radian lower ($\Delta R = 1$, $\Delta\Psi = 0.2$). In this case, the desired components of the movement vector, relative to the intrinsic α,β reference-frame can be established from the components. For convenience, assume that $R \ll r$. (This means that the hand stays close to the shoulder. As can be seen above, it greatly simplifies the given expression of the limb-movement tensor). Thus:

$$\bar{U}(\alpha, \beta) \approx \begin{pmatrix} \frac{1}{2r} & 1 \\ \frac{-1}{r} & 0 \end{pmatrix} . \bar{U}(R, \Psi)$$

$$= \begin{pmatrix} 0.05 & 1 \\ -0.1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 0.25 \\ -0.1 \end{pmatrix}.$$

Therefore, increasing α by 0.25 radian and changing β by -0.1 radian the hand is displaced by the desired vector, $\bar{U}$. Since the intended movement vector $\bar{U}$ was expressed in the extrinsic frame of reference and now the bodily components of the same vector are established, *a coordinated movement is made.*

When handling such problems mathematically one tends to employ, for reasons of convenience, orthogonal, rectilinear Cartesian frames of reference. However, even the α,β intrinsic reference-frame shows none of the above convenient features. E.g., by changing α or β one finds that the respective hand-displacements are almost never perpendicular to each other, indicating that the reference-frames are not orthogonal, but oblique. The displacements by α or β are not even straight lines: the system of coordinates is not rectilinear, but curvilinear. Accordingly, the expression of the limb-movement tensor in this non-Cartesian, oblique, curvilinear intrinsic reference-frame is position-dependent: the actual values of the components are different at different points of the hyperspace. Thus, the quantitative expression of the tensor-components is a set of non-constant quantities.* Therefore, in a coordinated action the transformation must always be available in an 'updated' form, corresponding to the actual position of the arm (cf. LLINÁS, 1974). While this requirement is serious, the overcompleteness of the intrinsic hyperspace relative to the external space poses the even more fundamental uniqueness problem which is analyzed below.

TENSORIAL OPERATIONS IN THE OVERCOMPLETE INTRINSIC CNS-HYPERSPACE

Consider a limb restricted to movements within a two-dimensional external space; however, let the limb be composed of *three* joints. With the corresponding angles of α, β and γ between them, any given movement vector can be produced by any one of an *infinite* number of combinations of α, β and γ alterations (cf. Fig. 2). However, every time a hand movement occurs there is just *one* actual implementation. It is also known that the cerebellum plays a key role in the process of arriving at this unique choice. The question therefore is: By what scheme can the cerebellum implement this coordination? As an example of the above, let us take the classical demonstration by HOLMES (1939) of the cerebellar dysmetria in a patient with a (left side) hemicerebellar lesion (see Fig. 2B). Here the intended displacements of the grossly simplified three-parameter system of the hand are given by two-dimensional vectors, expressed relative to the physical space (pointing to four corners of a square). As illustrated, a coordinated movement (a unique expression of the intended vectors in the higher than two-dimensional CNS hyperspace) can be implemented by the right normal side. In contrast, a coordinated movement cannot be executed with the left hand in accordance with the cerebellar disturbance produced by the unilateral lesion. The left hand displays what is known as a dysmetric motor action.

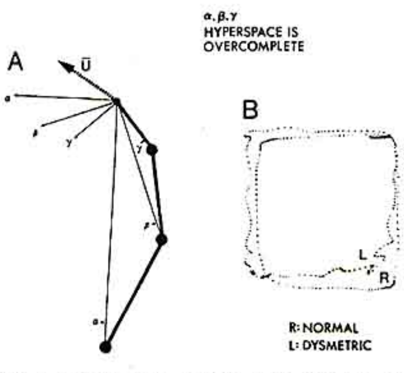

FIG. 2. The uniqueness problem of implementing an intended movement vector by a higher dimensional executor system. (A) A three-segment limb is to execute a two-dimensional intended vector $\bar{U}$, by virtue of changing the three coordinates of α, β and γ. The arrows at the joints indicate the infinitesimal changes of α, β, γ joint-angles. The α, β, γ system of coordinates at $\bar{U}$ shows the displacement corresponding to infinitesimal changes of the angles. (B) While the internal α, β, γ space is overcomplete, in reality the intended movement vectors *are* decomposed to an overcomplete number of components. Moreover, the used procedure is affected by cerebellar lesion: recorded hand movements, pointing to four corners of a square, shows satisfactory execution with the corresponding cerebellar hemisphere intact (R), while because of cerebellar damage, the movement of the left hand (L) shows haphazard movements known as dysmetria (B is after HOLMES, 1939).

* In special cases, as above, when limiting the movement to the shoulder vicinity, a careful selection of a polar coordinate system for the external space yields a tensor that *can* be simplified to a set of constants.