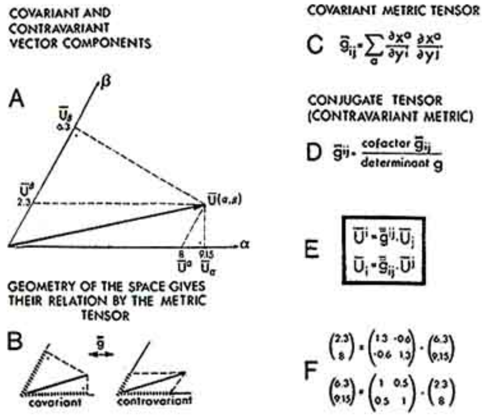

FIG. 3. Covariant and contravariant components of a vector, and their relation established by the geometry of the space of the vector. (A) The covariant components (lower index) can be established independently from one another, but they do not physically compose the vector. (These components, also called resolved parts are established by perpendicular projections.) Contravariant components (upper index) physically add up to the resultant, $\overline{U}$; however, they cannot be established independently from one another. (B) The numerical relation of the two sets of components is determined by the metric tensor which describes the geometry of the space. (C) Expression of the metric tensor in covariant form. (D) Expression of the contravariant metric tensor. (E) Relation of covariant and contravariant components, as determined by the metric tensor. (F) Numerical example of the relation of covariant and contravariant components shown in A and E.

In order to understand the manner in which the cerebellum implements this coordination, note that the displacement can be vectorially expressed both in the two-dimensional external space and in the three-dimensional intrinsic CNS hyperspace, i.e. the two-space can be regarded as a surface embedded into a three-space. Thus, the displacement, as an invariant line-element lies both in the embedded and in the embedding spaces. In order to analyze the vectorial expressions in these spaces the first question to be asked is whether these vectors are expressed by covariant or contravariant components.

Covariant and contravariant vector-components and the metric tensor

Some of the relevant features of covariant and contravariant vector-components and their transformation through the metric tensor, are summarized in Fig. 3. Given an arbitrary, oblique, two-dimensional frame of reference and a metric tensor, a single $\overline{U}(\alpha, \beta)$ vector may be expressed both by its covariant components $\overline{U}_i$ and by its contravariant parts $\overline{U}^i$. $\overline{U}^i$ are termed the 'physical components' since they, when added according to the parallelogram rule, *actually provide the resultant vector*, $\overline{U}$. In contrast as it is shown in Fig. 3, the covariant components do not have this feature, their physical sum not being equivalent to $\overline{U}$. On the other hand, covariant vectorial components have the important property that *a given vector component along one direction can be uniquely determined, independently of the total number of coordinate axes or of the direction of other axes.* Indeed, a covariant component is determined by taking the inner product with the unit vector in one coordinate direction, i.e. by establishing a perpendicular to the given axis. We call this *the principle of independence of covariant vector components.* Such a feature does not apply to the contravariant components, for which the establishing of any one component requires that all the other directions of the coordinates be known: that is to say, *the physical components, which are actually capable of generating the executing vector, and interdependent.*

The geometry of the hyperspace determines through the metric tensor, the relation of covariant and contravariant sets of components. This geometry can be expressed as shown in Fig. 3, in both covariant and contravariant forms. The $\overline{\overline{g}}_{ij}$ matrix is the covariant metric tensor, its conjugate, $\overline{\overline{g}}^{ij}$ is the contravariant metric tensor. Thus, the two sets of components are related through the following expressions:

$$\overline{U}^i = \overline{\overline{g}}^{ij}\overline{U}_j$$
$$\overline{U}_i = \overline{\overline{g}}_{ij}\overline{U}^j.$$

A numerical example given in Fig. 3 shows the actual calculations from one set of vectorial components to the other in the depicted case.

Our central tenet regarding the geometrical concept of coordination is that *the CNS must transform covari-*