ant components of a vector into their contravariant counterparts. If an intended movement vector is termed a 'movement image', the covariant features of this 'image' may be established independently along any given coordinate-axis. It also should be noted that there is no limit to the number of such covariant 'features', since the inner products, the perpendiculars, can be determined for any number of coordinate-axes. Note that establishing the inner products, i.e. 'taking the perpendicular' implies the use of the geometry of the embedded space. For example in the case of a two-dimensional external space and a three-dimensional CNS hyperspace the covariant components are established in the tangent plane of a surface that is embedded into the three-space. However, for the actual generation of the physical vector, the contravariant components are required. Thus, it is fundamental that the covariant set of components be transformable into the set of contravariants. This implies that, as proposed in this paper, CNS motor vectors reside in a hyperspace which is endowed with a metric tensor.

Covariant analysis and contravariant synthesis through a metric tensor

The above consideration permits the elaboration of a conceptual scheme which allows the decomposition of a vector, given in a lower dimensional space, into components expressed in an overcomplete hyperspace. The scheme is demonstrated in Fig. 4.

Given a three-segment limb, it was shown above that at any of its positions, the α, β and γ axes of coordinates establish a reference-frame that is overcomplete compared to a two-dimensional intended movement vector $\overline{U}$. Given the non-trivial assumption that the *intrinsic CNS hyperspace is endowed with a metric tensor*, the contravariant $\overline{U}^i$ set of components can be established by applying the metric tensor $\overline{\overline{g}}^{ij}$ to the vector $\overline{U}_j$. As shown in Fig. 4, such a set of contravariant components will actually create the required vector $\overline{U}$.

We consider this two-step operation to be a general principle applied by the CNS tensorial networks: (1) determining separately and independently from each other the covariant components ('features') of a movement image vector using the geometry of the embedded space, (2) transforming the covariant components into contravariant physical components through the metric tensor which establishes the geometry of the internal hyperspace. The first step, then, is a mechanism for analysis, since the characteristic features along different directions are 'measured' by establishing covariant components. However, these cannot create the vector. The second step is the mechanism for synthesis. Knowing the features of the intended result and the geometry of the space in the form of contravariant metric tensor, the actual resultant vector is provided by their product.

CEREBELLAR COORDINATION VIA A METRIC TENSOR WHICH ESTABLISHES THE GEOMETRY OF THE INTERNAL HYPERSPACE

In the above terms, a coordinated movement is seen as follows. An intended movement-vector is formulated in the CNS in reference to the embedded extrinsic physical three-space. This 'movement image' vector is resolved into covariant components along the coordinate-axes determined by the α, β and γ changes. The result of this analytic process is the set of features which the desired vector must possess. However, since the covariant components cannot be used to generate the desired vector, the covariant set of components must *undergo a transformation via the contravariant metric tensor of the hyperspace*. Since the cerebellum was represented by a tensor $\overline{\overline{\theta}}$ (PELLIONISZ & LLINÁS, 1979*a*), it is our proposal that *the cerebellar tensor acts as a contravariant metric which determines a geometry in the intrinsic hyperspace.* Therefore, the cerebellum would function by transforming the covariant components of the intended movements into contravariant components.

A computer-illustration of this principle is given in Fig. 5 using a limb composed of three segments moving within a two-dimensional plane. For a set of two-dimensional intended-movement vectors let us assume that the hand writes the letters 'OK'. In this case, the covariant components of the intended vectors can be established at every point of the writing movement. At each position of the limb one can determine perpendicular projections (inner products) to the momentary axes of α, β, γ reference-frame. If, however, these covariant components are being used to generate the displacement (changing α, β and γ according to the values of covariant parts), the resulting limb-movement is a haphazard set of displacements (Fig. 5B).*

In the next step (Fig. 5C) a symmetric matrix of 3 × 3 *constants* was set up to act as a contravariant metric tensor, establishing a position-independent intrinsic geometry of the α, β, γ internal hyperspace. Then the set of covariant components are transformed via this tensor to the contravariant components. Thus the two-dimensional intended vectors

---

* Note that such 'writing' resembles that ataxic, so-called dysmetric, movement of patients with cerebellar disease (cf. Fig. 2). In both Figs 2 and 5 the error in the direction and magnitude of the movement is *not* uniform: some intended vectors are less distorted—cf. last two segments of K—while some other vectors are grossly inadequate; see the connecting segment from O to K: This is because the sum of covariant components points into a direction that is sometimes similar to, and sometimes quite different from, the direction of intended vector: it depends on their relative directions. (It is only for the *eigenvectors* of the cerebellar tensor, for which the covariant-contravariant transformation does not make a change of the direction. Such eigendirections of the movement are not affected by cerebellar lesion.)