

case the difference of actions in a pair of antagonistic muscles, for example the lateral and medial recti, would constitute one coordinate axis.) While separate measurement of f^1 , f^2 and f^3 certainly simplifies matters with respect to \bar{Y} the tacit fundamental misconception need not be further belabored.

Indeed, it is generally felt that a direct, separable correspondence between particular eye muscles and particular eye movement directions is, at best, a simplification. This is emphasized by a scheme for the eye muscle 'cooperation' (e.g. CARPENTER, 1977, p. 133). He concludes that all eye muscles contribute to each component of motion; this implies that, in the present scheme, the off-diagonal elements of the tensor-matrix are indeed not zeros. Also, it should be noted that the weight from the i -th muscle to the j -th direction, which he denotes by g_{ij} (Fig. 7.23), could hardly come any closer to the identification of what this array represents. Still, the array of the matrix elements was shown apparently without recognizing that it is the matrix of the oculomotor tensor.

In tensorial terms, an important concern is the overcomplete character of the oculomotor space compared to the Euclidean external space. Here it must be assumed again that the extrinsic three-space is embedded into the six-dimensional internal hyperspace. This involves endowing the latter with an intrinsic geometry; by providing a contravariant oculomotor metric tensor. According to this scheme, the eye movements would be implemented, as above, in a two-step operation: the image vector of the intended eye movement would be decomposed first into covariant components (using the metric of the three-space) and then, via the matrix of the contravariant metric tensor of the oculomotor six-space, the covariant components would be transformed into contravariant components. The resulting motoneuronal firing frequencies to all six muscles would actually generate the intended movement vector. As was discussed in detail above and also in PELLIONISZ & LLINÁS (1979a), the ballistic movement of such displacements should show the individually characteristic trajectories that are determined by the geometry of the oculomotor space. Indeed, it is known that the saccadic eye movements, especially the oblique ones, do not usually move along straight lines, but show an interesting curved pattern (YARBUS, 1967; Figure 16; ROBINSON, 1972; VIVIANI, BERTHOZ & TRACEY, 1977). Since the position-dependent character of the tensorial expressions has been mentioned already, the above trajectories indicate that a metric tensor may be utilized to establish the geometry of a curved internal oculomotor hyperspace. Eye movement trajectories in the oblique direction would then correspond to the geodesics of this hyperspace.

Other more far reaching considerations may also be mentioned. Gaze is obviously generated not only by extraocular, but also by neck and other body muscles. In this case the head-acceleration is at least a six-dimensional vector and thus, the need for six-

acceleration sensors is immediately evident. Also, since the total tensorial system is, again, position and acceleration-dependent, the need for the otoliths and semicircular canals (i.e. additional position-sensors) is apparent. It is interesting to consider that with regard to the six-dimensional acceleration the six-dimensional oculomotor vector is *not* overcomplete. Note, however, that the *total* system is even more overcomplete than the one in the previous restricted case: the acceleration will evoke eye and neck movements and the dimensionality of this system is grossly overcomplete compared to a six-space and thus a metric tensor is required. This metric would provide a 'wired in' system of ratios of the neck and eye movements evoked by given accelerations. In addition, because of the overcomplete covariant decomposition of gaze as an intended movement vector; if one (or some) of the coordinate-axes were to be removed even during the movement in progress, target acquisition would still be possible with this scheme. This has been actually shown to be the case experimentally (BIZZI, 1974).

Some of the general implications of this tensorial viewpoint cannot be treated here; however, they will be discussed in a forthcoming publication. Suffice it to say at this juncture that while for a stationary body the external three-space is embedded into a stationary intrinsic hyperspace, in the case of acceleration of the body the external space must be embedded into the CNS hyperspace even though the two spaces accelerate relative to one another. This latter case involves a rather serious theoretical problem. A possible way to minimize this acceleration is, for instance, to stabilize the head relative to the external space (as in a pirouette, or in the free fall of a cat). This maneuver, i.e. using compensatory neck movements, could reduce or eliminate the accelerations of the head relative to the external space. In the simplest case, intended movement vectors concern a stationary body and a stable visual image. By stabilizing the head the above procedure of covariant analysis of intended movements could be retained even in case of acceleration. Then, since the neck movement vector provides information regarding how the body-space moves relative to the stabilized visual image, the covariant-contravariant transformation may be amended by this second transformation.

DISCUSSION

The geometry of brain function

Explaining brain function *more geometrico* requires that its function must be analyzed in terms of the abstract geometry of hyperspaces (PELLIONISZ & LLINÁS, 1979a). The advantages of a geometrical explanation of natural phenomena are clear: Geometrical methods describe the properties of natural systems more suitably than engineering methods which have been developed to describe man-made systems. Moreover, this view points directly to tensors as a formal and powerful approach to the global