analysis of brain function, since the concept of tensors extends beyond the particularities of vectors and matrices incurred by reference-frames. Reference-frame expressed vectors are commonly found in CNS networks and are usually described in terms of linear algebra. In fact, it is especially important to point out the fundamental features of the tensor concept, since linear algebraic matrix- and vector representations of neuronal networks are already familiar to readers. Early applications of vector algebra to CNS modeling can be found in WIENER (1948) and, most prominently, in the work of MCCULLOCH (1965), VON FOERSTER (1967), ASHBY (1967), GROSSBERG (1970), ANDERSON (1968), ANDERSON, COOPER, NASS, FREIBERGER & GRENANDER (1972), COOPER (1974), and KOHONEN, LEHTIÖ, ROVAMO, HYVÄRINEN, BRY & VAINIO (1977). As regards the cerebellum, multidimensional linear control systems have been mentioned by GREENE (1972).

In contrast to the above, the basic idea in the present set of papers (PELLIONISZ & LLINÁS, 1978; 1979*a*, *b*) is not that linear algebraic methods can be applied to CNS, but rather it is explicitly stated that these methods can be applied because *the brain is a tensorial system.*

Steps of abstraction: via vectors to tensors

To approach an understanding of global brain properties on the basis of available details two basic steps of abstraction are required:

(1) As previously postulated by many authors (see above), neuronal activity distributed over many elements is a *vectorial entity*, expressed in the reference-frame given by the neurons. Thus, the '*language*' *of the brain is vectorial.*

(2) The new consideration is that such vectors possess reference-frame invariant properties, thus they are *tensorial entities* (PELLIONISZ & LLINÁS, 1979*a*). This leads directly to questions regarding the properties of coordinate-free vectors, and beyond, to the space that contains them. Thus, the *brain as a system is tensorial*, i.e. it implements, by means of the neuronal circuits, tensorial solutions in the same sense that the lenses in the eye establish an object–image relation in a reference-frame-free manner.

As pointed out earlier (PELLIONISZ & LLINÁS, 1979*a*, pp. 344) the questions of orthogonality, the high dimensionality of CNS reference-frames and the homogeneous treatment of space-time we left untreated in our tensor network theory. Here we distinguish the two types of vectorial expressions which the cerebellar tensor θ, acting as a metric, transforms as the CNS uses a *non-orthogonal, overcomplete* system of coordinates. It is left, however, to a forthcoming paper to provide detailed explanation to the dynamic properties of cerebellar coordination. It will be shown that the motor vectors are, in fact, expressed in *space–time* components. Thus the concept of temporal 'lookahead' by Taylor series expansion, introduced in our earlier paper is incorporated into a unified concept of cerebellar space-time metric tensor.

Tensorial' responses are not separable into 'simple reflexes'

When considering a larger system (e.g. locomotion, or vestibulo-ocular reflex) it is difficult to view the total set of variables of the system in its entirety. Thus the tendency has been to 'slice' the whole into components that are investigated separately. This simplification is particularly clear when defining the properties of the so-called vestibulo-ocular reflex.

It is noteworthy that the analytical attitude (which, when unguarded, may lead to oversimplification) is deeply Cartesian. Descartes, by decomposing complex systems into their components, laid the foundations of analytical geometry: he noted that the position of a point can be characterized by an orthogonal set of coordinates. Moreover, it was this approach that led him to develop the biological concept of 'reflexes'. By analogy from analytical geometry, brain function was thought to be characterizable by a set of separate, simple components—the reflexes. Thus the 'simple reflex' became prevalent for a three-hundred-year period. However, the idea in time has increasingly suffered from an overemphasis on the components, as separable entities. This continued even after the central figure in reflex physiology, SHERRINGTON (1906), strongly emphasized that 'a simple reflex is probably a purely abstract conception'. Re-establishing the entity of the whole vector itself, rather than considering only its components, is not conceptually easy nor is it experimentally attractive; dealing with separate components is both mathematically and experimentally much simpler. Given one component, the concepts and techniques developed for the analysis of a single scalar variable (like a control feedback loop, widely used in engineering) may appear to be applicable. Such an approach, e.g. applied for the vestibulo-ocular 'reflex', seemingly obviates the problem of treating the brain as other than a set of separate reflexes. *Nevertheless the separation of the properties of a system into parts is, in general, incorrect if complete analysis is required. If the latter is sought, a tensorial approach can treat the global entity itself, rather than its separate parts.* To further emphasize this point one may quote HEAVISIDE (1925): '. . . for general purposes of reasoning the manipulation of the scalar components instead of the vector itself is entirely wrong'.

Coordination by covariant–contravariant transformation by a metric tensor

The two-step procedure outlined above, i.e. establishing covariant components of an intended 'image' vector in an embedded space and then transforming these 'features' to the contravariant components of the same vector in an embedding space (by means of a contravariant metric of the latter), may be considered the central conclusion of this paper. The transformation requires a contravariant metric of the internal