space which we believe is supplied in the case of the motor system by the cerebellar matrix θ. At present, we are only concerned with the subspace of movement vectors; other CNS subspaces will be treated elsewhere. In view of the studies of a metric employed in binocular vision (without using tensor analysis; cf. LEIBOVIC, BALSLEV & MATHIESON, 1971), it is evident that the full utilization of the tensorial approach not only in motor, but also in the visual and other sensory systems, should yield further insights. As already indicated, several properties are as yet unexplored concerning the cerebellar matrix θ in serving as a metric tensor. For example, it is morphologically unknown (*a*) whether θ is a square matrix or (*b*) whether it is symmetrical or (*c*) non-singular. Also, it was observed that the geometry is position-dependent, i.e. the CNS hyperspace is curved. Whether the curvature of the space is perturbed by the vectorial climbing fiber system, as well as whether the curvature is an intrinsic property of the inherent CNS space or is associated with the reference-frame, will be considered elsewhere.

In short, the generation of a movement (i.e. the execution of an intended vector) is produced by the covariant–contravariant vector transformation by the cerebellar tensor. Utilization of the covariant components of an intended movement is shown here, by computer modeling, to produce an ataxic movement with all of the characteristics of dysmetria and tremor generally found in cerebellar malfunction.

Covariant analysis and contravariant synthesis

Beyond coordination, we assume that *the covariant feature analysis and contravariant component synthesis is applicable whenever a space is embedded into a hyperspace*. Therefore, it is suggested that this paradigm may be fundamental not only in motor but also in CNS sensory systems. Indeed, the principle of independence in establishing separate perpendicular projections of a vector seems particularly appropriate to sensory systems. Sensors work independently of each other; even ablation of some may not alter the functioning of the remainder. Thus a sensation as a vectorial entity is decomposed by them into covariant rather than contravariant components. From a non-tensorial point of view a similar conclusion was derived by MCKAY (1978) who emphasized that 'the *analysis of covariation* must be an ubiquitous function of the perceptual nervous system'.

Clearly, while our tentative conclusion is that the cerebellum functions as a metric tensor, and that such conception allows a formal description of the parallel and distributed properties of the central nervous system as a motor coordination device, the properties of the internal CNS hyperspace must be further explored. We consider the present paper mainly as an effort to provide insights into developing a geometrical theory of brain function.

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