Neuroscience Vol. 7, pp. 2949 to 2970, 1982
Printed in Great Britain

0306-4522/82/122949-21$03.00/0
Pergamon Press Ltd
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SPACE-TIME REPRESENTATION IN THE BRAIN. THE CEREBELLUM AS A PREDICTIVE SPACE-TIME METRIC TENSOR

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The concept of space-time representation in the brain is redefined using tensor network theory. We make the following suggestions. (a) In order to deal with the external world, the brain embeds the external space-time continuum into a high dimensional internal space. External space-time events are represented within the CNS in overcomplete, inherently oblique, reference frames where space and time information is detected as a continuum over each coordinate axis. (b) The central nervous system may be seen as imposing a geometry on this internal hyperspace in such manner that neuronal networks transform inputs in a metric tensor-like manner. (c) In order to coordinate movements the cerebellum acts as a predictive motor space-time metric which allows the establishment of coincidences of goal-directed movements of limbs in space-time with external targets.

Space-time: dogma and redefinition

Understanding of brain function is often limited by the tacit acceptance of concepts known to be basically inappropriate. An example is the tenet that the brain, in its internal workings, utilizes space-time reference frames similar to those used in classical mechanics.

The fundamental problem may be briefly stated. The process of locating and intercepting moving objects by an animal (a coincidence of the interceptor and a target) is generally described as a coordinated sensorimotor act. While both sensory and motor functions relate to events taking place in external space-time, a concise description of the manner in which space and time information are inter-related in the CNS has not been clearly formulated. One of the reasons for this deficiency appears to be the usage of separate frames of reference for 'space' and 'time'. Such frames are used, for instance, in the 'Newtonian' space-time representation, where x, y, z coordinates of a point represent the location in the Euclidean space, while a separate t, established by a clock, serves as a coordinate of the time-point of the event. The Newtonian representation is quite satisfactory in classical mechanics. For instance, if the space-time coordinates of an ordinary moving object are measured, light can be used as a means to establish simultaneity. Indeed, when filming a moving object, the consecutive numbers, labeling the individual frames, can provide a common time measure t for all x, y, z spatial coordinates. Alternatively, in flash photography the light 'freezes' time such that simultaneous space coordinates (all belonging to the same t) can be established for all points represented in the photograph.

However, the utilization of separate systems of coordinates for space and time hinges upon the concept of simultaneity. Since within the brain there is no 'instantaneous' simultaneity agent comparable to the light, the classical usage of separate space and time coordinates is inapplicable in the case of describing the inner workings of the CNS.

We indicated an alternative approach in two preliminary communications.[22,23] In this paper, we elaborate the proposed concept of space-time representation in the CNS, based on Tensor Network Theory.[19,20,21] Our central assumption is that the brain works in a tensorial manner: that is (a) the function of the CNS (the activity of neurons) is expressed vectorially, using various frames of reference; (b) a given physical entity (e.g. movement) may be represented by different vectorial expressions (e.g. covariants and contravariants); (c) when such vectors are assigned to the same physical entity, by definition they are related to one another tensorially. Because external entities, such as a coincidence, are independent of the coordinate system in which they are described, we will refer to them as *invariants*.

Space-timing and cerebellar theories

The necessity of formulating a concept of not just 'timing' but 'space-timing' in the CNS becomes evident when considering cerebellar function. Indeed, space-timing is particularly important with respect to the coordination of rapid and precise muscular movements such as a saccadic eye displacement.[27] The mode of command by which space and time information is handled by the CNS is essential to the understanding how, e.g. saccadic eye movements are mation is handled by the CNS is essential to understanding how, e.g. saccadic eye movements are made.

Hitherto, our understanding of the space-time representation in the brain has been conceptually based