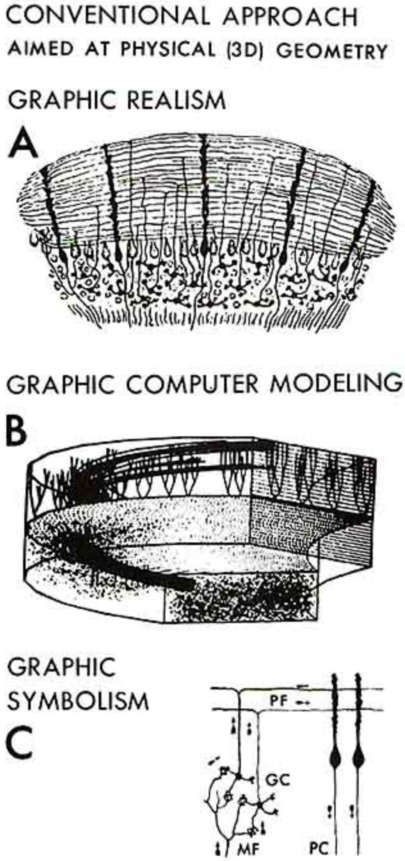

Fig. 1. Levels of abstraction in the conventional representation of neuronal networks, describing structural geometry only. A: a circuitry diagram of the cerebellar cortex (from Fig. 26 in ref. 25). B: computer model of the cerebellum (modified from Fig. 6 in ref. 24). C: a graphic symbol of the cerebellar neuronal 'loop' (from Fig. 104 in ref. 25). MF: mossy fibers, GC: granule cells, PF: parallel fibers, PC: Purkinje cells.

FUNDAMENTALS OF THE TENSORIAL APPROACH TO SPACE-TIME REPRESENTATION IN THE BRAIN

Basic notions on the tensorial representation of invariant events

Our approach is based on the consideration that a space-time coincidence is a phenomenon, inherently independent of whatever system of coordinates may be arbitrarily assigned to it, that we call an *invariant*. We further assume that in the CNS an activity-pattern of $f_1, f_2, \ldots, f_n$ firing frequencies over a set of n neurons can represent, internally, the same invariant that is represented externally by Cartesian space and separate time coordinates.

The distinction between a Newtonian approach and the one introduced here becomes evident as follows. An event-point is usually described in a Newtonian frame of reference by a mathematical vector (an ordered set of real numbers) $M(x, y, z, t)$. In the CNS the same invariant may be represented by another mathematical vector $F(f_1, f_2, \ldots, f_n)$, which is a different ordered set of quantities. The concept put forth here relies on the fact that the two vectorial descriptions (M and F) are equally appropriate, i.e. neither is, *a priori*, pre-eminent. Given that both mathematical vectors are assigned to the same invariant, it follows that M and F are tensorially related to one another. This consideration provides a basis for a tensorial treatment of the space-time representation in the brain.

From a mathematical standpoint the tensorial approach views the brain in terms of abstract geometry. The intrinsic functional geometry of the CNS hyperspace (the multidimensional space over the points F) is an internal representation of the external physical geometry (the latter existing over the set of points M). Thus, the adequate mathematical approach is that of related geometries: one in the four-dimensional *physical* space externally (usually represented by Euclidean geometry), and another, structural geometry within the CNS networks. Still another is the *functional geometry* in the CNS hyperspace, a largely unknown, but certainly not Euclidean, geometry.

The technical steps of geometrically featuring 'brain vectors'[23] and some experimental approaches to reveal the coordinate systems inherent in brain function[30] are described elsewhere.

A concise example of the application of tensor network theory to CNS: the cerebellum as a space-metric

The CNS can be viewed, *more geometrico*, in several ways and at many levels of abstraction in each. Figures 1 and 2 provide contrasting views of representing the geometries involved in structuro-functional properties of the CNS. The conventional approach is directed at the structural geometry of the brain (Fig. 1). The descriptions in Figs 1A, B, C, aim at establishing the system of spatial relations among the physical components, i.e. of the different types of nerve cells in the network. The alternative tensorial view of representing not only the structural, but also the functional geometry of neuronal networks is given in Fig. 2. The tensorial approach also *relates* the structural and functional geometries to one another, since it treats them by a method that is capable of unifying structuro-functional descriptions. Thus, Fig. 2 serves as a comprehensive introduction of the application of tensor concept to brain function in this paper.

A most conventional way of representing a neuronal assembly is shown by Fig. 1A. Such a descriptive approach is intuitively clear but has serious limita-