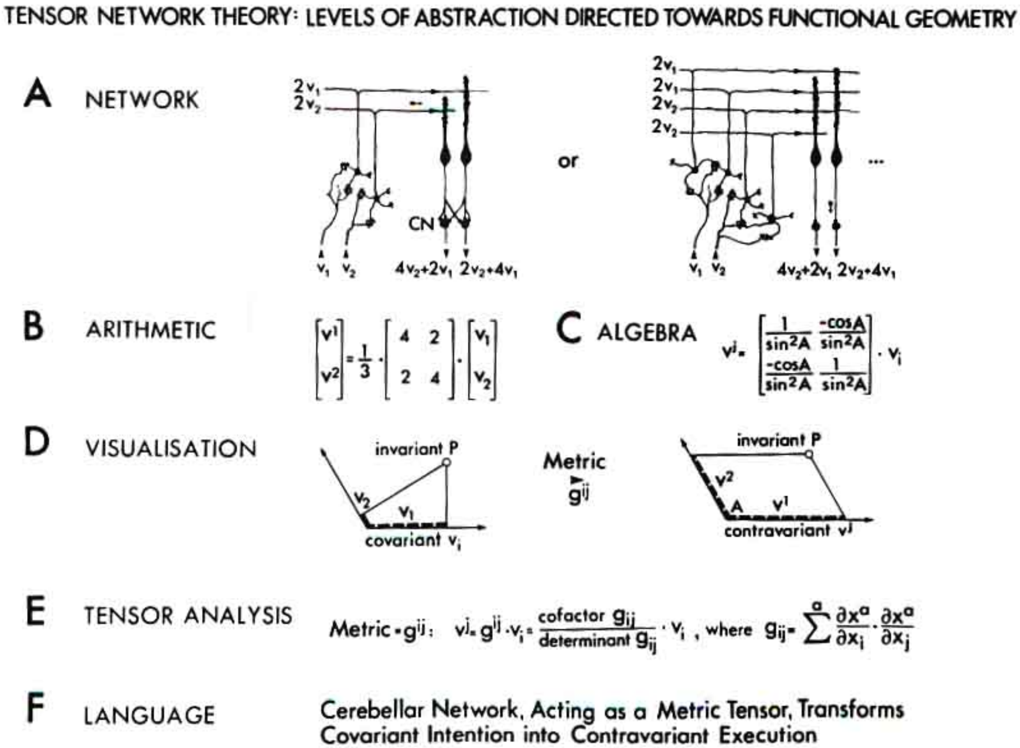

Fig. 2. Levels of abstraction in the representation of the functional geometry of neuronal networks by tensor network theory. A concise demonstration of how a set of neuronal networks (A) can perform a transformation to be described by a general tensor, such as the metric (E, F). The 2-dimensional Euclidean visualization of the vector transformation (D) is a diadactic convenience, utilizing the heuristic power of graphics. A: Two of the infinite number of possible network implementations of a quantitatively identical transformation from a multiple input to a multiple output. CN: cerebellar nuclei. B: A numerical matrix expression of the transformation *via* the networks shown in A. A scaling factor of 1/3 is included to make possible a common quantitative handling from A to D. C: Expression of the transformation-matrix for a whole class of two-dimensional, rectilinear oblique reference-frames where the axes are at a variable angle A to each other. D: visualization of the functional purpose of the particular transformation implemented by the networks. The transformation implements a covariant–contravariant conversion of one kind of vectorial expression into another, both vectors being assigned to the same invariant physical entity P. E: reference-frame invariant expression of contravariant metric tensor: a general formula for covariant–contravariant transformation of vector components for all and any systems of coordinates. F: verbal expression of the tensorial interpretation of cerebellar function.

tions as it does not address the question of function of the network, nor that of quantitative characteristics. A second type of representation is typified by the use of computer models[18,24] (Fig. 1B). While such models make it possible to address quantitative properties of large networks, they do not necessarily address questions of function, nor provide a generalizable abstraction of the structural properties of the CNS circuits. This latter goal is achieved by a third kind, a symbolic representation (Fig. 1C), which provides an easily drawn realistic logogram that often stands for the properties of the total neuronal network.[25] In spite of its usefulness as a graphic abbreviation of the cerebellar structure, this particular graphic symbol does not provide an adequate concept of the functional organization of the structure. The input-output neuronal loops suggest that the fundamental aspect of CNS function relates to its serial organization, rather than to a parallel processing through interconnected network.

In contrast to the conventional approaches featured in Fig. 1, Tensor Network Theory aims both at describing the structural geometry and at understanding the functional geometry of neuronal networks. Furthermore, these goals can be achieved at several levels of abstraction (Fig. 2). The abstraction has to commence with a quantitative representation of the network. Thus, starting from the stage shown in Fig. 1C, the reflex arc logogram is made first into a network where the transformations from a multiple input to a multiple output are quantitatively traced. A pair of such parallel networks is shown, in a schematic manner, in Fig. 2A. Both circuits implement a quantitatively (and, as we point out later, qualitatively) identical transformation of a v_i input vector into an output vector. Note that exactly the same transformation can be implemented not only by the two networks depicted in Fig. 2A, but by an infinite number of other particular network variations. With this approach, the group of all such networks capable