

As suggested earlier,^{21,22} sensorimotor coordination can be conceptualized as a transformation from a sensory to a motor vector within general (non-orthogonal) systems of coordinates (see Fig. 3). Components of sensory information are established independently of one another; that is, if sensory information is expressed vectorially, it is of the covariant type. Motor execution, in contrast, is expressed as the resultant of physical components, i.e. a vectorial expression of motor execution is of the contravariant type. Therefore, a transformation from sensory- to motor-information is a covariant-contravariant conversion. Thus, any sensorimotor system capable of expressing an invariant, both from a sensory and a motor aspect, must contain implicitly or explicitly at least one contravariant metric which transforms one type of vector into the other.

A simple model of a sensorimotor system containing such a metric tensor is provided in Fig. 3A. Suppose that the goal of this system is to locate and intercept a moving target, which at time t is at the physical location P . If the target were a sound source, a 'sensory' system could be implemented by two linear arrays of microphones along the directions of the axes, measuring the orthogonal projections; i.e. covariant v_1 and v_2 components of the OP distance from the origin to P . A 'motor' system could be a mechanical device consisting of two rods that can be advanced to any length determined by the v^1 and v^2 contravariant motor coordinates. Assume, as the simplest case, that the sensory and the motor frames of reference are identical, i.e. both constitute the same angle A ($A = 120^\circ$ throughout this paper). A simulation of a biological equivalent of such a simple system is shown in Fig. 3A, featuring two identical frames of reference, a sensory (auditory) and a motor (limb). It is obvious that biological systems utilize reference frames different from that shown in the model (e.g. biology is vastly more complicated). The point of this rudimentary model is to serve as a demonstration of an extremely important fact. Even if the sensory and motor systems have only two dimensions, and identical frame of reference, the sensory information cannot be directly applied to execute motor actions. If the sensory components that locate the source were transmitted unchanged to the motor system, 'dysmetric' or 'ataxic' performance would result (Fig. 3B). Indeed, while the movement of the target P is circular, the executed movement of P' is an ellipse. Thus, the sensory v_i components cannot be equated with the v^j motor components; a covariant- to contravariant metric is required for this transformation to be correct.

Later, we will show that following our scheme most sensorimotor systems may contain at least two additional complicating factors which refer to the differences between the sensory and motor coordinate frames: (i) their arrangement (the directions of the axes in the sensory and motor coordinate systems may not be identical) and, (ii) their dimensionality

(the number of coordinate axes in the sensory and motor reference frames may be different).

This is the case, for example, in the gaze-stabilizing system of CNS; when the body moves, the eyes and the head move in a compensatory manner. The sensory and motor frames in this system are different (even if only the vestibular and oculomotor frames are considered). In the fully general case of gaze-stabilization, the motor apparatus includes the neck muscles as well as the eye muscles; thus, in such a system the sensory and motor frames are vastly different both in the number and the direction of the coordinate-axes. The symbolic scheme in this paper is intended as a model and a suggestion how to treat tensorially any such sensorimotor system that employs different input-output frames (such as vestibulo-ocular reflex, gaze-stabilizing system, etc.).

The transformation matrix needed to convert covariant sensory information into contravariant motor signals can be implemented by a 'neuronal network' in Fig. 3C. With this network serving as a space-metric, the circling movement of the target is not only properly detected, but an identical movement is faithfully executed. However, the transformations shown throughout Fig. 3 are synchronous; i.e. it is assumed that the vector components are free of temporal delays. Since such synchrony is an assumption that is in conflict with the presence of a variety of conduction-times in signal transmission and processing, a problem is raised which will be considered below in regard to additional features of the metric.

UNIFIED SPACE-TIME REPRESENTATION IN THE CNS

The coordinates of the space-time continuum in a sensorimotor system with non-uniform conduction times

Conduction time differences are usually disregarded when global brain function is modeled. That is, one assumes that the system is synchronous in spite of the fact that different neuronal channels have different conduction velocities and lengths which ultimately result in different conduction times. While in experimental analyses of particular subsystems of the CNS delays are usually taken into account, they are interpreted as temporal phase-differences of the neuronal responses, usually to sinusoidal test-functions. Such an approach leaves the development of a conceptually coherent interpretation of space-time representation not only lacking, but almost hopeless (cf. Fig. 4).

In the terms introduced in the scheme of Fig. 3, the delays mean that each of the v_i coordinate components represents not only the space position of the target, but also the moment in time when each particular position occurred. Thus, the function of the space-metric shown in Fig. 3 has to be expanded to account for time delays; i.e. the network must function as a *space-time metric tensor*. This is shown in Fig. 4.

In this figure the delayed covariant components of