

expressed by  $g^{j\Omega T}$ , that is, the cerebellar space-time tensor changes the lower  $i$  space index to an upper  $j$  and the lower time index  $\Omega$  into an upper  $T$  index.

It would be desirable to identify in the CNS those networks that perform 'implicit' unified sensorimotor space-time metric functions (as in Figs 6 and 7), and those that more closely correspond to the expanded, explicit sensory and motor metrics (as in Fig. 5).

Before doing so, it must be pointed out that the reason for the development of an explicit metric in the CNS appears to be economy in the number of neuronal components. Assume, for example, that 100 neurons are required for a single lookahead-module (a reasonable assumption; cf. Fig. 1 in ref. 20). Then, the separable explicit scheme (cf. Fig. 5) would require 'only'  $100 \times 2 \times n$  'differentiator' neurons, since lookahead-modules are built into the  $n$  input lines and into the  $n$  output lines. In turn, in the implicit scheme (cf. Fig. 6) the necessary number would be  $100 \times n^2$ , since all of the  $n \times n = n^2$  connecting lines of the matrix contain lookahead-modules. The difference between  $2n$  and  $n^2$  becomes very significant if  $n$  is very large. Thus, in contrast to one's intuitive expectation, a conceptually 'compacted' implicit scheme requires an enormously higher number of neuronal elements than an explicit scheme which performs the identical function.

For example, if the space-time metric in question is the cerebellum and the differentiating neurons are the Purkinje cells, then using an implicit network would require one million Purkinje cells in the case of a one hundred dimensional sensorimotor system. By contrast, only twenty thousand Purkinje cells would suffice for a separable, explicit sensorimotor metric. Likewise, for a larger system (e.g. with ten thousand-dimensions) these figures would be ten billion Purkinje cells versus two million.

The one hundred dimensional hyperspace may approximate an amphibian (e.g. frog) cerebellum, while the ten thousand dimensional sensorimotor system may be that of, for example, a cat. The requirement of twenty thousand Purkinje cells for the frog cerebellum is well within the range of morphological reality,<sup>7</sup> whereas one million Purkinje cells is far too large a number. Similarly, a number as large as ten billion Purkinje neurons in the cat cerebellum can be ruled out, while two million Purkinje cells are well within the range of established facts.<sup>17</sup> Thus, according to our view, in very high dimensional full sensorimotor systems, an explicit metric is more likely to be implemented than the implicit variety.

In contrast, for a rudimentary sensorimotor system that acts only on a dozen muscles (e.g. the vestibulo-ocular reflex) even an implicit sensorimotor network is within morphological reality, since  $n = 10$  would require only a small nucleus composed of about ten thousand neurons.

#### General tensorial architecture of sensorimotor systems

The possibility that separable 'explicit' sensory and

motor metrics are present in sensorimotor systems is a reminder of the fact that sensory and motor coordinates usually differ both in dimensionality and directionality.

A difference in the directionality of the coordinate axes in the sensory and motor systems poses no problem if the number of coordinate axes in the motor system is equal to or less than the dimensionality of the sensory system, i.e. the motor hyperspace is a projection of the sensory hyperspace. In such a case, all that is needed is an additional projection-matrix,  $p^{jk}$ , which transforms the motor vector expressed in the sensory-type frame (upper index  $j$ ) into the motor frame of reference, where the components have an upper index  $k$ . Such a  $p^{jk}$  can be inserted into the scheme featured in Figs 5-7 by placing it after the metric transformation, so that the overall connectivity matrix becomes  $g^{ij} \cdot p^{jk}$ , (an  $i \times k$  matrix). In this case, the metric transformation and the change of reference-frame can be implemented in one transformation by a single input-output network. The  $d^k$  'motor' lookahead, of course, has to equal the delay in the  $k$ th output of the motor system. As above, the contraction of such an explicit metric into a single (implicit) metric can be implemented in the same manner as in the case where its sensory and motor frames of reference are identical. In the compacted 'implicit' network a 'lookahead module' has to be incorporated into each of the  $i \times k$  connection to yield a temporal prediction of:

$$d^{ik} = d_i + d^k.$$

In contrast to the above, the total sensorimotor system cannot be implemented with a unified single matrix (as in Figs 6 and 7) if (a) a specific requirement necessitates an explicit space-time metric tensor and/or (b) there is an increase of the dimensionality from the sensory to motor systems (the motor system has a higher degree of freedom than the sensory). In the latter case, the motor hyperspace is not a projection of the sensory space, but the sensory system (which embeds the physical three-space) itself becomes an embedded space in the motor hyperspace.

Separate sensory and motor space-time metric tensor may be a biologically desirable feature in itself for reasons other than the above-mentioned difference in reference frames or because of circuit economy. A fundamental reason for sensor and motor separation is that any CNS subsystem that has to reach realistic conclusions about the external invariant itself must incorporate a separate metric tensor. Indeed, *neither a covariant nor a contravariant vector alone* can express such parameters of the invariant as distance. However, it is known that distance, for example, can easily be expressed by the inner product of the two different types of vectors:  $D^2 = v^i \cdot v_i$ . Here, the simultaneous availability of both the co- and contravariant vectors implies the availability of the metric tensor, since  $v^i \cdot v_i = g^{ij} \cdot v_j \cdot v_i$ . Given that the metric tensor summarizes all geometric relations, its presence allows the