extraction of other invariants, e.g. angles, geodesics, etc.

Evidently, some sensorimotor systems work in a manner such that at times only the sensory part is active, producing, on its own, a biologically meaningful decision relating to the invariant. Such is the case, for example, when a visual judgement is made of distance, without the necessity of the motor system (other than the extraocular eye muscles) being simultaneously engaged in this sensory process. The biological purpose of such a sensory system is to incorporate, within itself, an adequate (geometrically homomorphic) internal representation of external invariants.

Using the frog as an example, it is a fact that a jump establishing a space-time coincidence does not follow, automatically, from every visual message; it occurs only when the space-time event (the fly's location) is within range. In our terms, it must be decided that the physical invariant, the distance between the fly and the frog, is within the range of possible jumps. The comparison of such invariants even in this simple case is further complicated by the fact that they are 'distances' of space-time event-points, involving not just space but also temporal aspects such as speed. As a consequence, a jump to a non-moving or an overly fast moving target will not occur even if the target is within spatial range.

The above considerations suggest that sensory perception requires the building of a sensory metric, separable from the motor system, that transforms covariant sensory vectors into contravariant sensory vectors expressed in a sensory reference frame. Indeed, the existence of such a sensory metric was suggested, prior to the tensorial approach, for vision,[8] and color perception.[12] We suggest and will further elaborate below that the sensory covariant–contravariant transformations serve as precursors to the generation of motor intention vectors, also in the sense of permitting fundamental sensory decisions on whether motor action is appropriate.

A general tensorial architecture of a sensorimotor system can be outlined now, based on the above considerations. In a general case the sensory frame of reference is different from the motor frame, the latter being allowed to have any number of dimensions (including a greater number of axes in the motor frame than in the sensory system of coordinates). Another general feature is the possible existence of a separate sensory space-time metric and a separate motor space-time metric. A sensorimotor scheme with the above general features is shown in Fig. 8. Because of the differences in sensory and motor frames, the circuitry schematics require several different vectorial expressions attributed to a single external space-time event-point. In the scheme of Fig. 8, each vector is not only conceptually and geometrically well-defined, but each also corresponds to a biologically well-circumscribed CNS function.

A minimum of four basic vectorial expressions are required since two different frames of reference are used, and in each frame there is a covariant and a contravariant version of the vector. Thus, the principal vectorial expressions of the general functions of a sensorimotor system are:

(1) *Reception vector*, defined here as a covariant expression in the sensory frame of reference of the external invariant. In such a vector, the components are derived directly from the invariant and established independently of one another. Such vector, however, contains the information in a 'scrambled' manner, e.g. not only the spatial information is represented along all axes, but these vector components also contain a temporal 'blur'.

(2) *Perception vector*, defined here as a contravariant expression in the sensory frame of reference of the external invariant. The existence of both the covariant- and contravariant vectors implies a sensory metric tensor. This contravariant metric provides the basis for building a homomorphic internal model of the external world. If the geometry of the internal representation resembles the external realities, then judgements based upon the internal geometry will provide successful guidance for manoeuvering within the system of relations existing in the external world.

(3) *Motor intention vector*, defined here as a covariant expression in the motor frame of reference of the external invariant. This expression is a necessary precursor of motor execution vector, since the components of this vector are the ones that can be obtained from the sensory perception vector by a procedure called covariant embedding (taking the inner products of the perception vector with another vector). The latter we refer to as sensorimotor proprioception vector, defined here as a covariant expression of the unit vectors of the motor frame, in the sensory frame of reference (see inset at lower right of Fig. 8). The procedure of the covariant embedding yields a mathematically unique set of components for the intention vector even if the motor frame has a higher dimensionality than the sensory frame. Obtaining the intention vector by inner product requires only multiplying and summing networks, which are matrix operations that the CNS networks can perform easily (as shown below). The intention vector, although a representation of the external invariant, cannot be directly utilized to generate the motor action because of its covariant character.

(4) *Motor execution vector*, defined here as the contravariant expression in the motor frame of reference of the external invariant. The components of this vector, by definition, will be capable of physically assembling the external invariant. However, in order to obtain the contravariant from the covariant intention vector, a motor metric (a contravariant metric) has to be available within the motor system of the CNS.

With the use of these function vectors of the CNS, the scheme of Fig. 8 provides an explanation of the functioning of the sensorimotor system from sensory