reception to motor execution, using four different levels of abstraction; the 2D Euclidean illustration, verbal description, tensor notation and network repreresentation. Since this figure is an expansion of the previous figures of this paper, the explanation can rely largely on the earlier schemes, with only few differences. First, Fig. 8 uses different visual symbols for the neuronal elements of the sensory lookahead-modules than the ones used in Figs 5–6. The neuronal elements in Figs 8 and 9 were redrawn from appropriate parts of the original drawings (Figs 102, 275, 323, 330) of Ramón y Cajal.[25] *The reason for such representation is that we deem it absolutely necessary to demonstrate that the conclusions of a theoretical abstraction, such as those of the tensor theory, are not inconsistent with the fundamental morphological realities of existing neuronal networks.* Just as abstraction in Figs 1 and 2 started with realistic representation of networks, theory has to conclude in yielding exactly such neuronal networks. Another feature in Fig. 8 is that the sensory system is augmented by a simple auxiliary network that multiplies the corresponding covariant and contravariant sensory component-pairs and sums their products, thereby taking the $D^2 = v_i.v^i$ inner product. If the threshold of this inhibitory neuron is set at $D^2 = r^2$ (r being the jump limit), then the sensorimotor transduction is blocked whenever the target is not within range.

Strictly speaking, the auxiliary circuit of sensorimotor blocking is not necessary for the functioning of the sensorimotor scheme. However, its inclusion serves two purposes: (a) to demonstrate the simplicity of networks making internal geometrical decisions on external invariants and (b) to raise the issue of inner product implementation, especially since it is needed also for the operation of covariant embedding, the next step of the scheme. Indeed, a sum of products may not be produced by a single neuron in the CNS. A technically significant implementation of an inner product is worth mentioning in this paper. As shown in the covariant embedding part of Fig. 8, neurons with logarithmic input–output characteristics sum up the inputs and the result drives a neuronal output with exponential characteristics. With such an 'embedding module', a local network consisting of only three neurons can circumvent the requirement for multiplicator neurons.

The covariant embedding part of Fig. 8 produces a motor intention vector that goes through a network, representing the 3×3 matrix of the motor metric tensor, and then through the 'motor lookahead' modules to provide the contravariant execution components. Each lookahead-module is symbolized in Figs 8–9 by only two Purkinje cells.

In short, a network as in Fig. 8 can transform a $v_{\beta i}$ *covariant sensory reception vector, expressed in two-dimensional sensory frame and consisting of temporally lagging asynchronous components, into a* u^{m1} *contravariant motor execution vector, expressed in a differently arranged, higher dimensional motor frame, with each component of the asynchronous output having a temporal lead* d^m.

The network shown in Fig. 8 satisfies the theoretical requirements from a sensorimotor scheme, and correlates well with several neuroanatomical features of the CNS. The network illustrated in Fig. 8 can also be depicted by a general simplified scheme of the spatial organization of CNS networks as, for instance, in an amphibian (Fig. 9). Comparable classical schemes of the neuronal networks of the regions of the CNS suggest that particular (morphologically distinct) brain regions may be engaged primarily in sensory functions, while others perform mostly motor functions. Indeed, while even the simplest blueprint of the global functioning of the CNS lies further in the future, it seems reasonable to suggest that while a function corresponding to a motor space-time metric is probably implemented by the cerebellum, the superior colliculus may well serve, in lower vertebrates, as a sensory space-time metric.

DISCUSSION

Several general comments regarding transformations *via* tensor networks can be offered. From a *technical point of view*, a covariant embedding (either of the external points into the sensory space or of the perception-points into the motor space) yields a unique set of covariant components, where the individual components are established independently of one another, regardless of the dimensionality of the embedding system. That is, an embedding space is free to have any number of dimensions; it may be vastly overcomplete compared to the embedded space. In Fig. 8 for example, the physical point of the target-location can be embedded into not just two, but any dimensional sensory space. Likewise, in establishing the covariant intention vector, the motor hyperspace may have any number of coordinate axes, totally independent of the sensory frame. The process of covariant embedding thus provides a solution to the vexing problem of how the CNS arrives at unique solutions even though it has an overly high degree of freedom.

Interestingly, the covariant-contravariant scheme of a covariant embedding followed by a metric transformation also reconceptualizes the old notion of functional reliability of neuronal networks achieved by means of structural redundancy.[35] The orthogonal projection-character of the embedding covariant components (independent of one another), together with the overcompleteness of the embedding hyperspace, yields an increased reliability of the contravariant expression. Firstly, because of the independence of their establishment, errors do not accumulate, the accuracy of measuring one covariant not affecting the accuracy of establishing the other. Secondly, in the case of an overcomplete system, a loss or drastic alteration of some covariant components will not necessarily result in a proportional distortion of the contravariant out-