

put, since some of the errors cancel each other out through the interconnections of the metric. The numerical distortion produced by a damage of the covariant input depends on the metric, e.g. on the size of the off-diagonal elements it contains. The further these elements are from zero, the more reliable the system is, but the higher the number of input-output connections which are required for the morphological implementation of the matrix. Thus, while conceptually different, an overcomplete tensorial system resembles the classical scheme of redundant organization in the sense that there is a trade-off between reliability and structural economy in both.

The overcomplete covariant embedding does not only apply to sensorimotor systems. Indeed, as shown in Fig. 9, covariant embedding followed by metric transformation leads to a cyclic scheme of CNS function where the number of dimensions may be arbitrary in each subsystem, and covariant and contravariant vectorial expressions (shown in blue and red, respectively) alternate throughout the scheme.

Methodological implications. This paper conjoins the tenets of our two preceding papers on the tensorial approach to brain function. The first paper introduced tensor network theory and proposed a temporal predictive feature of cerebellar neuronal network.²⁰ The second established the thesis that the cerebellum acts as a metric tensor transforming the covariant intention vectors into contravariant execution vectors.²¹ The present paper provides a synthesis; it proposes that instead of acting only as a space-metric, the cerebellum acts as a metric tensor of the unified space-time manifold working with temporally extrapolated space-time coordinates. This unification is significant also from a technical point of view, as it merges the mathematical devices used in those previous papers. This merge helps the answering of an obvious methodological question regarding the paradoxical adequacy of 'linear' tensor analysis in treating nonlinearity-laden systems.

Indeed, tensor analysis is applied most often to linear problems. However, as is well known, it is quite appropriate in nonlinear theories such as relativity. Equally prominent is the applicability of tensors in the description of nonlinear (only locally linear) tensions in inhomogeneous elastic bodies. In particular, in this paper 'prediction' is suggested to compensate for the temporal delays. Going forward or backward

in time are not linear features in the common use of the term. The overall function of the network can still be characterized by a fundamentally linear metric function, because one kind of nonlinearity (look-ahead) compensates for the other (delay). In fact, linearity in any physical system never exists in the true mathematical sense; no physically implemented 'straight line' is linear upon close examination. Thus, in the CNS, just like in any other physically implemented system, the question is not whether linear methods of description are applicable, but to what extent they are usable.

The conceptual unification of space and time handling in CNS suggested by this paper reaches further than the simple merging of methods. Basically, the issue is how to relate our understanding of the geometrical structure of CNS to our understanding of the physical space-time geometry. In mechanics, Newton, by his utilization of distinct space and time frames, established a separation, rather than a merging, of the concepts of space and time. At a philosophical level, the space-time schism (in the sense of attributing separate existence to these two parts of one entity) was completed late in the 18th century by Kant, who held space to be an *a priori* concept of mind.

The synthesis of the notions of mechanics with those of brain function started relatively recently. While Wiener³⁸ and McCulloch¹³ mentioned that the Newtonian frame is not the only one applicable to living systems, a systematic elaboration of Newtonian space-time representation in the brain had not been offered until Braitenberg's paper.¹ He characterized his approach thus: "the structure of the cerebellar cortex is viewed in the same spirit in which we would analyze an unknown machine".² It is true, that the CNS is a 'machine', in the sense that it imposes an order on the moving parts of the musculoskeletal system; it is also evident, that such movements are externally describable by classical Newtonian mechanics. It does not follow, however, that the inner workings of the CNS utilize the same Newtonian mechanics used in the description of the movement. Indeed, since Newtonian mechanics is only one among the several different types of mechanics (e.g. Newtonian, quantum, relativistic) in an unknown machine, both the implementing device (the mechanism) and the set of rules that govern such device (the mechanics) must be taken as unknown.

Fig. 9. Circuitry layout, a tensorial 'blueprint' of the model of an amphibian brain. This network is identical to the one shown schematically in Fig. 8, only the layout of the network is different in order to demonstrate that the tensorial interpretation is fully consistent with the conventional descriptions of the neuroanatomical realities. The correlation of the conventional and the tensorial approaches must be made evident in order to ensure that the descriptive and abstract representations do relate to one and the same reality. Thus, the neuronal elements shown symbolically in this figure have been redrawn from appropriate figures in ref. 25 (see text). Such tensorial scheme (beyond being a realistic representation of neuronal networks) can actually perform, *via* covariant embedding procedures followed by space-time metric transformations, the fundamental operations of a two-to-three-dimensional sensorimotor network. Pathways in this scheme carry the neuronal information vectorially (covariants shown in blue, contravariants in red). The cortical circuitries perform tensor transformations. For further explanation, see text.